Diode-Based VCAs: real-time, explicit, nonlinear models using the **Wright Omega** function

Real-Time Virtual Analog Modelling of Diode-Based VCAs

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Introduction

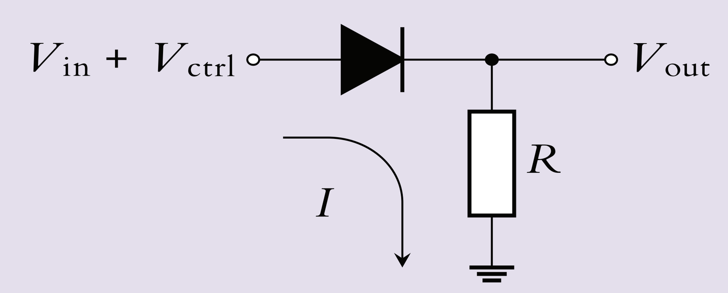
- Voltage-Controlled Amplifiers (VCAs) are fundamental building blocks of signal processing
- Digital VCAs are usually perfect multipliers
- Many analog VCA implementations exist, each with **unique distortion and dynamics**
- Some historically-relevant audio devices use **semiconductor diodes** as variable-gain elements
- Diodes exhibit **dynamic resistance** based on DC operating point
- Diode-based VCAs have **distinctive sonic qualities** due to their nonlinearity
- Did not find any prior literature analyzing or modelling diode-based VCAs

Methodology

The paper presents digital models of three distinct diode-based VCA topologies, which were created using the following steps:

1. Construct a simplified analog circuit prototype by isolating the variable-gain section
2. Derive an implicit equation relating the input signal, gain control parameter, and output signal using Kirchhoff's circuit laws and the Shockley diode equation
3. Use the Wright omega function to convert the implicit equation into an explicit function
4. Reparameterize the equation to decouple the model from physical component values, using separate parameters for bias and sharpness
5. Compute the small-signal gain expression by taking the partial derivative of the output signal with regard to the input signal
6. Invert the small-signal gain expression to parameterize the model's gain

1.



2.

$$I_S \exp\left(\frac{V_{in}+V_{ctrl}-V_{out}}{\eta V_T}\right) - I_S - \frac{V_{out}}{R} = 0$$

3.

$$V_{out} = \eta V_T\,\omega\left(\frac{V_{in}+V_{ctrl}}{\eta V_T} + \omega^{-1}\left(\frac{I_S R}{\eta V_T}\right)\right) - I_S R$$

4.

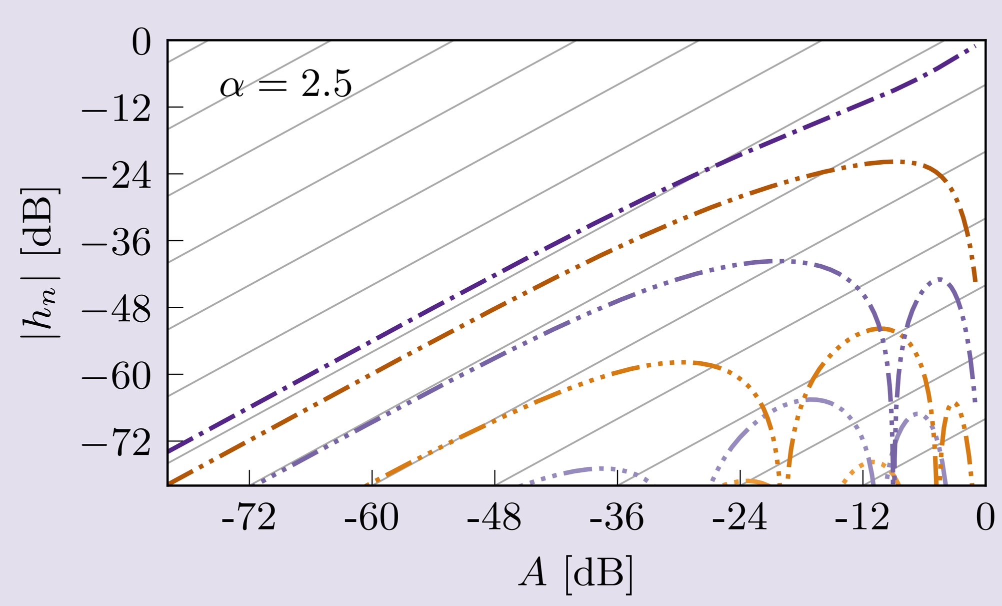
$$y(x) = \alpha^{-1}\omega(\alpha x+\beta) - \alpha^{-1}\omega(\beta)$$

5.

$$A := \lim_{\alpha\to 0}\frac{\partial y}{\partial x} = \frac{\omega(\beta)}{\omega(\beta)+1}$$

6.

$$\beta(A) = \omega^{-1}\left(\frac{A}{1-A}\right)$$

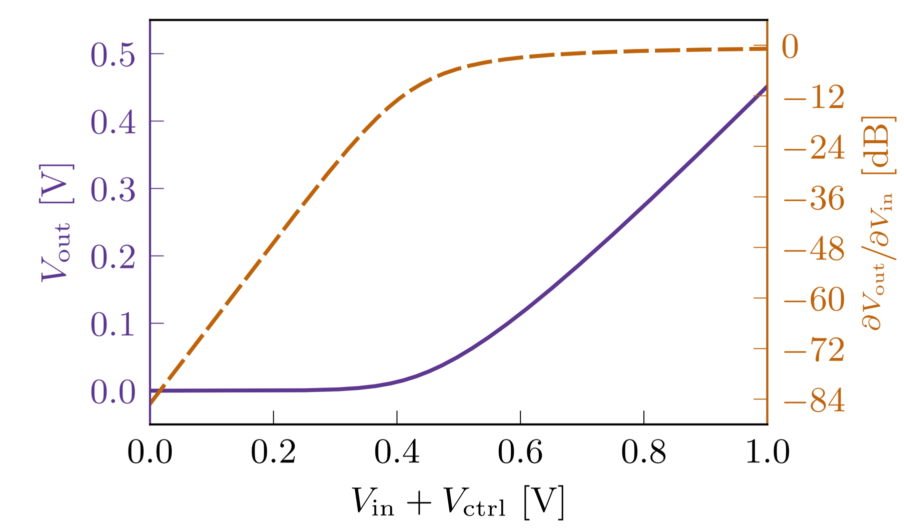


Relative magnitudes of each harmonic generated when applied to a sinusoidal input signal for different gains

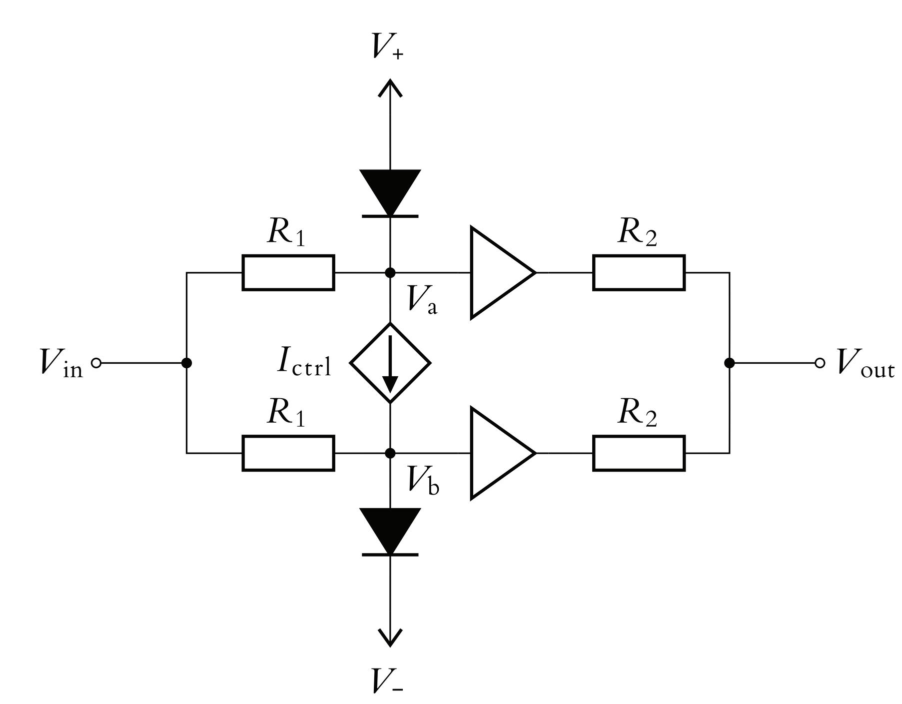
Results

- Distortion dependent on gain parameter: THD inversely proportional; odd and even harmonics clustered in lobes
- Response nearly indistinguishable from SPICE simulation
- No iterative algorithms; suitable for real-time evaluation

Diode-Resistor Signal Sweep

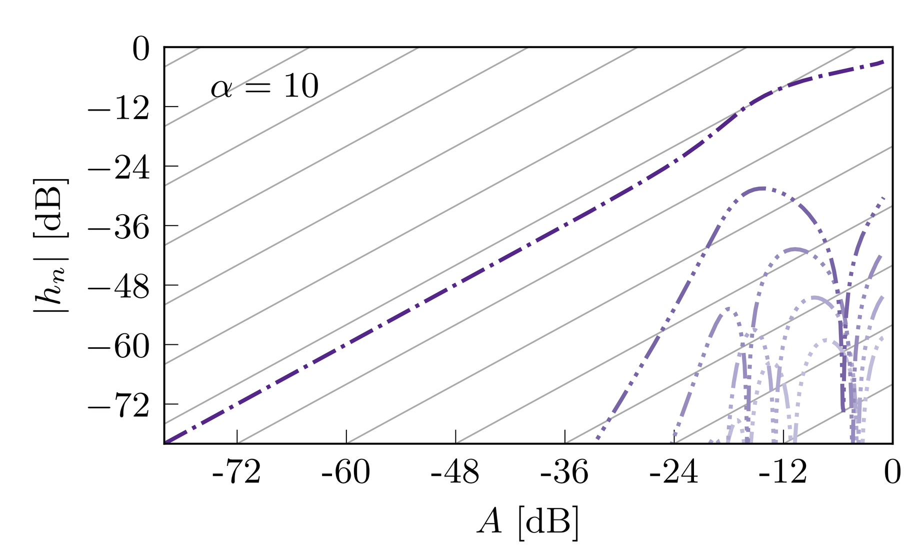


Selected Topological Variation



$$y(x) = x - \frac{1}{2}\alpha^{-1}\omega(\alpha x+\beta) + \frac{1}{2}\alpha^{-1}\omega(-\alpha x+\beta)$$

$$\beta(A) = \omega^{-1}\left(\frac{1-A}{A}\right)$$



Future Work

- Additional circuit topologies
- Diode-based VCA as a subcomponent of a larger system (e.g. compressor, ring modulator, dynamic filter)
- Family of bias-controlled VCAs:

$$y(x) = \alpha^{-1}f(\alpha x+\beta) - \alpha^{-1}f(\beta)$$

$$A := \lim_{\alpha\to 0}\frac{\partial y}{\partial x} = f'(\beta)$$

$$\beta(A) = [f']^{-1}(A)$$

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Scan for full paper, interactive demo, and additional resources

https://corianderpines.org/pub/